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Abstract Title: A generalized Radon transform in imaging

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A generalized Radon transform in imaging

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We consider a generalized Radon transform that integrates functions over quadric surfaces (ellipsoids, hyperboloids) with centers on a smooth connected hypersurface. These operators appear in sonar, radar and seismic imaging as forward operators which map the image to the data. The goal is to invert them by applying the back-projection operator. By doing this, artifacts appear. We determine their strength and try to diminish their effect on imaging.

We use microlocal techniques to study the properties of these operators, their composition, and the artifacts. The plan is to study the singularities (i.e., when these maps drop rank) of the right and left projections of the canonical relation associated with these operators. The canonical relation describes the relation between the singularities of the data and the singularities of the image. The nature of these singularities (i.e. folds and cusps) determines the artifacts and the reconstruction of the image. The quadric surfaces are described by a diagonal, invertible matrix with real entries. We show that the signature of this matrix establishes the singularities of the projections. If the matrix is positive or negative definite, we prove that the singularities are folds and if the matrix is indefinite then, cusps singularities are present. In these cases, we show that the artifacts are very strong, and we are looking for ways to diminish their strength. This work extends on a previous paper on these transforms where the projections do not drop rank, thus no singularities were present.

Reference:

James W. Webber, Sean Holman, and Eric Todd Quinto. Ellipsoidal and hyperbolic Radon transform microlocal properties and injectivity. J. Funct. Anal., 285(8): Paper No. 110056, 30, 2023.